Chapter 7

7.1 Let T be the outcome of a roll with a fair die.

a)

Outcomes	1	2	3	4	5	6
Probability Mass	1/6	1/6	1/6	1/6	1/6	1/6
Cumulative Dist.	1/6	2/6	3/6	4/6	5/6	1

b)
E[T] = 1 ×1/6 + 2 × 1/6 + 3 × 1/6 + 4 × 1/6 + 5 × 1/6 + 6 × 1/6 = 3.5
Var(T) = 1/6× (1 – 3.5)^2 + 1/6× (2 – 3.5)^2 + 1/6× (3 – 3.5)^2 + 1/6× (4 – 3.5)^2 + 1/6× (5-3.5)^2 + 1/6× (6-3.5)^2

= 1/6(6.25 + 2.25 + .25 + .25 + 2.25 + 6.25) = 2.917

7.2 X is a random variable with the following distribution

P(X=-1) = 1/5
P(X=0) = 2/5
P(X=1 = 2/5

a)
E[X] = -1 × 1/5 + 0 × 2/5 + 1 × 2/5 = 1/5

b) Y = X^2

P(Y = 0) = 2/5
P(Y = 1) = 3/5
E[Y] = 0 × 2/5 + 1 × 3/5 = 3/5

c)
The change of variable formula for expectation is
(-1)^2 × 1/5 + 0^2 × 2/5 + 1^2 × 2/.5 = 1/5 + 0 + 2/5 = 3/5 which agrees with b)

d)
Var(X) = 1/5 × (-1 – 1/5)^2 + 2/5 × (0 – 1/5)^2 + 2/5 × (1 – 1/5)^2
= 1/5 × 1.44 + 2/5 × 0.04 + 2/5 × .64 = 0.552

7.9 U is a random variable with distribution U(a, b)

a) The probability density function F(u) = 1/(b – a) if a ≤ u ≤ b, 0 otherwise.
The anti-derivitave of u × F(u) is
u^2 / (2 × (b-a))
So the definate integral is (b^2 – a^2) / (2 × (b-a) = (b – a)(b + a)/2(b – a)
= (a + b)/2.

b)
To find the Var(U) we integrate $1/(b - 1) \times (u - (a + b)/2)^2$ from a to b.
This gives $(a + b)^2 / 12$

7.13
For any random variable X,
$0 \leq \text{Var}(X) = E[X^2] - (E[X])^2$
Re-arranging terms gives
$E[X]^2 \geq (E[X])^2$.

7.14
We choose an arbitrary point from a square with vertrices at
(2,1), (3,1), (2,2), and (3,2). Let A be the random variable which is the area of the triangle formed by the chosen point and the points (2,1) and (3,1)

This area will be given by the half the product of the (y-1) and the distance from (2,1) to (3,1) (which is of course 1), where y is the y-coordinate of the chosen point.
Thus $A = (Y-1)/2$. Y is uniformly distributed between 1 and 2. Its expected value is 1.5
Thus $E[A] = E[Y] / 2 - ½ = 1.5 /2 - .5 = ¼$